Practice Lesson 20 Area of Composed Figures

**Prerequisite: Separating a Figure to Find the Area**

Study the example showing how to decompose a shape to find its area. Then solve problems 1–7.

**Example**

Bev creates the pattern shown. Bev’s pattern is made up of 6 triangles, 2 whole hexagons, and 2 half hexagons. She wants to find the area of one of the hexagons.

To find the area of one hexagon, Bev divides a hexagon into 1 rectangle and 2 congruent triangles. Then she finds the area of each shape.

- **Area of triangle**
  \[ \text{Area of \( \frac{1}{2} \) hexagon} = \frac{1}{2} \times \text{area of one \( \frac{1}{2} \) hexagon} \]
  \[ = \frac{1}{2} \text{Area of \( \frac{1}{2} \) hexagon} = \frac{1}{2} \times 12 \text{ square units} \]
  \[ = 6 \text{ square units} \]

- **Area of rectangle**
  \[ \text{Area of rectangle} = (\text{length} \times \text{width}) \]
  \[ = (6 \times 6) \text{ square units} \]
  \[ = 36 \text{ square units} \]

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**Key**

- **B** Basic
- **M** Medium
- **C** Challenge

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**Practice and Problem Solving**

**Unit 4 Geometry**

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Lesson 20

Dividing a Figure to Find Its Area

Study the example showing how to find the area of a composite figure. Then solve problems 1–7.

Example

A STOP sign is a polygon with 8 equal sides. Find the area of the STOP sign.

First, draw lines to divide the figure into polygons whose areas you know how to find. Look for polygons that are identical and label them with the same letter. Sketch one of each of the shapes and label its dimensions. Then find their areas.

Area of square A = \(bh = (12.4)(12.4) = 153.76\)

Area of rectangle B = \(bh = (12.4)(8.8) = 109.12\)

Area of right triangle C = \(\frac{1}{2}bh = \frac{1}{2}(8.8)(8.8) = 38.72\)

Total area = \(153.76 + 4(109.12) + 4(38.72) = 745.12\)

The area is 745.12 square inches.

In the example, why were Area B and Area C multiplied by 4 but Area A was not?

There are 4 Area Bs and 4 Area Cs, but there is only 1 Area A.

How do you know the dimensions of rectangle B?

Possible answer: The length, 12.4 in., is marked on the figure. The width can be found by subtracting 12.4 from 30 and dividing by 2.

How were the dimensions of right triangle C found?

Possible answer: The height, 8.8 in., is marked on the figure. The base can be found by subtracting 12.4 from 30 and dividing by 2.

Solve.

M 4 Show how you can divide the figure at the right into polygons whose areas you know how to find.

Possible answer is shown on the figure.

M 5 Find the total area of the figure in problem 4.

Possible work: Area of parallelogram = \(bh = 10\frac{1}{2} \cdot 7\frac{1}{2} = 78\frac{3}{4}\) square inches

Area of figure = \(2 \cdot 78\frac{3}{4} = 157\frac{3}{4}\) square inches

M 6 Pedro says that the solution to problem 5 is 78\(\frac{3}{4}\) square inches. How did he get that answer?

Possible answer: Pedro forgot to multiply by 2 to account for the two parallelograms.

C 7 Refer to the figure at the right.

a. Show how you can divide the figure at the right into polygons whose areas you know how to find. Then write an equation that represents the total area of the figure, \(T\).

Possible answer is shown on the figure.

Possible equation: \(T = A + 2B + 2C\)

b. Sketch each kind of figure and label its dimensions.

c. Find the total area of the figure.

Show your work.

Area of square A = \(2(2) = 4\)

Area of rectangle B = \(2(3) = 6\)

Area of right triangle C = \(\frac{1}{2}(2)(2) = 2\)

\(T = A + 2B + 2C\)

\(T = 4 + 2(6) + 2(2) = 20\)

Solution: \(20\) square centimeters
Lesson 20

Solving Problems Related to Area

Study the example showing how to use what you have learned to solve problems related to area. Then solve problems 1–7.

Example

Ben is paving a parking lot with the shape shown. How can he find the area of the parking lot?

One way is to complete a rectangle around the parking lot and then subtract those areas that are not part of the parking lot. This is what Ben decides to do.

Area of parking lot - Area of large rectangle = Area A + Area B

What is the area of the large rectangle? Show your work.

11,250 square yards; Area = \( \ell w \) = \((150)(75)\) = 11,250

Find the dimensions of rectangle A. Then find the area of rectangle A. Show your work.

The length is 40.5 yards and the width is \( \frac{1}{2}(75 - 25) \) = 25, or 25 yards. Area = \( \ell w \) = \((40.5)(25)\) = 1,012.5 square yards

Explain how you can find the dimensions of triangle B. Then find the area of triangle B.

The base is \( 150 - 100 - 40.5 = 9.5 \) or 9.5 yards; the height is \( 75 - 2(25) \) = 25 yards. Area = \( \frac{1}{2}bh \) = \( \frac{1}{2}(9.5)(25) \) = 118.75 square yards

Find the area of the parking lot. Explain your calculation.

Area of parking lot = 11,250 - 2(Area A) - (Area B)

= 11,250 - 2(1,012.5) - 118.75 = 9,106.25 square yards

I subtracted 2 times Area A because there are 2 identical rectangles labeled A, as well as the area of triangle B from the area of the large rectangle.

Solve.

M 1 A side of a barn has the shape shown. Use subtraction to find the area of the side of the barn. Draw any lines on the diagram that will help explain your work.

Show your work. Let \( A \) = area of side of barn, \( S \) = area of square, and \( T \) = area of triangle.

\[
A = S - T
\]

\[
= 100(100) - 2(8.5)(25)(25)
\]

\[
= 10,000 - 625 = 9,375
\]

Solution: The area of the side of the barn is 9,375 square feet.

M 2 Jasmine solved problem 5 using the diagram at the right. She says that an expression for the total area is \( (100)(75) + (50)(25) + \frac{1}{2}(25)(25) \). Do you agree? If so, show that her expression is equivalent to the one you found in problem 5. If you disagree, explain her error.

Show your work. Jasmine added Area A, Area B, and Area C. But there are two identical triangles with Area C. She should have multiplied \( \frac{1}{2}(25)(25) \) by 2.

Solution: Disagree; Jasmine should have multiplied the area of triangle C by 2.

M 3 Find the area of the STOP sign using subtraction. Draw any lines needed on the diagram to explain your thinking.

Show your work. Find the area of the 30-in. by 30-in. square.

\[
A = \frac{1}{2}bh \Rightarrow \frac{1}{2}(8.8)(8.8) = 38.72
\]

Area of STOP sign = 900 - 4(38.72) = 745.12

Solution: 745.12 square inches
Lesson 20

Area of Composed Figures

Solve the problems.

1. Complete the equations by writing A, B, C, or D in each box to represent the area of each region. The large rectangle is D.
   - Area of trapezoid = \( A = \frac{1}{2}(b_1 + b_2)h \)
   - Area of trapezoid = \( B = \frac{1}{2}(b_1 + b_2)h \)
   - Area of trapezoid = \( C = \frac{1}{2}(b_1 + b_2)h \)
   - Area of trapezoid = \( D = \frac{1}{2}(b_1 + b_2)h \)

2. The dimensions of a small flag are shown. Which of the following expressions does NOT represent the area of the region shown in white?
   - A \((2)(6) - \frac{1}{2}(2)(3)\)
   - B \((4)(6) - \frac{1}{2}(4)(3) - (2)(3)\)
   - C \((3)(2) + \frac{1}{2}(3)(4)\)
   - D \((6)(2) + \frac{1}{2}(3)(4)\)

3. Perry drew this design on graph paper. He says that because the design is made of 2 identical squares he can find the area of his design by multiplying the area of one square by 2. Do you agree? If so, explain why. If not, explain Perry’s error. Draw lines on the design to show your thinking.

   Disagree; Possible answer: Because the two squares overlap, Perry needs to subtract the area of the part that is underneath the top square from twice the area of 1 square.

4. Write the area of each region of the design in square units.
   - Area A: \( 3 \)
   - Area B: \( 18 \)
   - Area C: \( 2 \)
   - Total Area D: \( 2 \)
   - Total Area: \( 25 \)

5. Abe plans to lay tiles on the areas shown. A case of tiles costs $40 and covers 15 square feet. How much does Abe need to budget for the tiles?
   - A $360
   - B $400
   - C $440
   - D $480

   Claire chose B for the answer. What did she do wrong?

   Solution: Abe needs about 10.8 cases of tiles. Claire multiplied $40 by 10 instead of 11.

6. What percent of the total area of the flag is white?

   Show your work.

   Total area = \( \ell w = (8)(6) = 48 \)
   - Area of 2 black triangles = \( 2 \cdot \frac{1}{2}bh = 2 \cdot \frac{1}{2}(4)(3) = 12 \)
   - Area of 2 blue triangles = \( 2 \cdot \frac{1}{2}bh = 2 \cdot \frac{1}{2}(6)(2) = 12 \)
   - White area = \( 48 - 12 - 12 = 24 \)
   - Percent white = \( \frac{24}{48} = 50\% \)

   Solution: 50% of the flag is white.