Lesson 18
Problem Solving with Angles
Name:

Prerequisite: Find Unknown Angle Measures

Study the example problem showing how to use subtraction to find unknown angle measures. Then solve problems 1–6.

Example

In most desk chairs, people sit at a 90° angle. The angle that is best for your back is 135°. What is the difference in degrees of these two sitting angles? You can use a diagram to help you better understand the problem.

100° sitting position
135° sitting position

difference between angles

The equation $135° - 90° = a°$ represents this situation.
The difference $135° - 90°$ is $45°$, so $45° = a°$.

You can also use a protractor to help you. Start at 0° on the inside scale of the protractor. Count to 90° on the protractor. How many more degrees do you need to count to get to 135°?

45°

Some desk chairs have as small as a 75° sitting position. What is the difference in degrees between sitting in this chair and its smallest setting and sitting at 135°?

$135° - 75° = 60°$

Solution: The difference is 60°.

You can also use a protractor to help you. Start at 0° on the inside scale of the protractor. Count to 90° on the protractor. How many more degrees do you need to count to get to 135°?

45°

Some desk chairs have as small as a 75° sitting position. What is the difference in degrees between sitting in this chair and its smallest setting and sitting at 135°?

$135° - 75° = 60°$

Solution: The difference is 60°.

A wheelchair ramp for a business cannot be steeper than 5°. A similar ramp for a home can be 10°. What is the difference in degrees of these two ramps? Explain.

Solution: The difference is $5°; 10° - 5° = 5°$.

Randi had surgery on her knee. Right after the surgery, she could bend her knee only 30° from a horizontal position. After six weeks of physical therapy, she can bend it 108°. How many more degrees can she move her knee after therapy? Explain.

Solution: Randi can move her knee 78° more after therapy: $108° - 30° = x°$, so $x° = 78°$.

Mario places a laptop on a desk. He adjusted the screen so that the screen is at an angle of 81° with the desk. What is the measure of the unknown angle? Explain.

Solution: The unknown angle is 99°.

Mansi shines a beam of light on a mirror. The angle between the beam and the mirror is 25°. The beam reflects off the mirror at an angle of 25°. Find the measure of the unknown angle.

Solution: The unknown angle is 130°.

Solve.

B 1
A wheelchair ramp for a business cannot be steeper than 5°. A similar ramp for a home can be 10°. What is the difference in degrees of these two ramps? Explain.

Solution: The difference is $5°; 10° - 5° = 5°$.

M 3
Randi had surgery on her knee. Right after the surgery, she could bend her knee only 30° from a horizontal position. After six weeks of physical therapy, she can bend it 108°. How many more degrees can she move her knee after therapy? Explain.

Solution: Randi can move her knee 78° more after therapy: $108° - 30° = x°$, so $x° = 78°$.

M 5
Mario places a laptop on a desk. He adjusted the screen so that the screen is at an angle of 81° with the desk. What is the measure of the unknown angle? Explain.

Solution: The unknown angle is 99°.

C 4
Mansi shines a beam of light on a mirror. The angle between the beam and the mirror is 25°. The beam reflects off the mirror at an angle of 25°. Find the measure of the unknown angle.

Solution: The unknown angle is 130°.

Key

B Basic  M Medium  C Challenge
Lesson 18

Use Supplementary and Vertical Angles

Study the example showing how to use supplementary and vertical angles to find angle measures. Then solve problems 1–8.

Example

A teacher draws a stop sign. She extends \( \angle ADB \) and \( \angle BDC \), as shown. Write an equation that you can use to find the value of \( x \).

\( \angle ADB \) and \( \angle BDC \) are supplementary angles, so the sum of their measures is 180°. This means that you can use the equation \( 3x + (8x + 15) = 180 \) to find the value of \( x \).

**B 1**

How do you know that \( \angle ADB \) and \( \angle BDC \) are supplementary?

The angles form a straight line.

**M 2**

Solve the equation \( 3x + (8x + 15) = 180 \) to find the value of \( x \).

**Show your work.**

\[ 3x + (8x + 15) = 180 \]
\[ 11x + 15 = 180 - 15 \]
\[ 11x = 165 \]
\[ x = 15 \]

**Solution:** \( x = 15 \)

**M 3**

Use your answer to problem 2 to find the measures of \( \angle ADB \) and \( \angle BDC \).

**Show your work.**

\( \angle ADB = (3x)^\circ = (3 \cdot 15)^\circ = 45^\circ \)
\( \angle BDC = (8x + 15)^\circ = (8 \cdot 15 + 15)^\circ = (120 + 15)^\circ = 135^\circ \)

**Solution:** \( \angle ADB = 45^\circ; \angle BDC = 135^\circ \)

**B 4**

What is the measure of \( \angle ADE \)? Justify your answer.

135°; \( \angle ADE \) and \( \angle BDC \) are vertical angles, so their measures are the same.

**Vocabulary**

- *supplementary angles*: two angles whose measures add up to 180°.
- *vertical angles*: congruent angles formed when two lines intersect.

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Solve.

In the diagram, \( \overline{BC} \) and \( \overline{DC} \) are extended as shown. Use the diagram for problems 5–7.

**M 5**

Find the value of \( x \).

**Show your work.**

\( \angle BCE \) and \( \angle ECF \) are supplementary angles.

\[ (10x + 15) + 5x = 180 \]
\[ 15x + 15 = 180 \]
\[ 15x + 15 - 15 = 180 - 15 \]
\[ 15x = 165 \]
\[ x = 11 \]

**Solution:** \( x = 11 \)

**M 6**

Use the diagram and the value of \( x \) to find the measures of \( \angle BCE \) and \( \angle ECF \).

\( \angle BCE = 125^\circ \)

\( \angle ECF = 55^\circ \)

**M 7**

What is the measure of \( \angle BCD \)? Justify your answer.

The measure of \( \angle BCD = 55^\circ \). \( \angle BCD \) and \( \angle ECF \) are vertical angles, so their measures are the same.

**C 8**

Suppose you know that two angles, angle \( A \) and angle \( B \), are supplementary. The measure of angle \( A \) is 5 times the measure of angle \( B \). Find the measures of the angles.

**Show your work.**

Let the measure of angle \( B \) be \( x^\circ \). Then the measure of angle \( A \) is \( 5x\circ \).

Because the angles are supplementary, \( x + 5x = 180 \).

So \( 6x = 180 \), and \( x = 30 \), and \( 5x = 5 \cdot 30 = 150 \).

**Solution:** \( \text{The measure of } \angle B = 30^\circ \). \( \text{The measure of } \angle A = 150^\circ \).
Lesson 18

Use Complementary and Vertical Angles

Study the example showing how to about complementary and vertical angles to find angle measures. Then solve problems 1–8.

Example

In the diagram, \( \angle MON \) and \( \angle LOM \) of rectangle \( LMNO \) are extended as shown. Write an equation that you can use to find the value of \( x \).

Because \( \angle MON \) and \( \angle LOM \) are complementary angles, the sum of their measures is 90°. This means that you can use the equation \( 2x + (7x + 9) = 90 \) to find the value of \( x \).

B 1 How do you know that \( \angle MON \) and \( \angle LOM \) are complementary angles?

LWNO is a rectangle, so \( \angle MON \) and \( \angle LOM \) together form a right angle.

M 2 Solve the equation \( 2x + (7x + 9) = 90 \) to find the value of \( x \).

Show your work.

\[
2x + (7x + 9) = 90 \\
9x + 9 = 90 \\
x = 9
\]

Solution: \( x = 9 \)

M 3 Use your answer to problem 2 to find the measures of \( \angle LOM \) and \( \angle MON \).

The measure of \( \angle LOM \) = 18°

The measure of \( \angle MON \) = 72°

B 4 What is the measure of \( \angle POQ \)? Justify your answer.

18°; \( \angle LOM \) and \( \angle POQ \) are vertical angles, so their measures are the same.

Vocabulary

complementary angles: two angles whose measures add up to 90°.

M 5 Find the value of \( x \).

Show your work.

\( \angle DOT \) and \( \angle TOU \) are complementary angles.

\[
4x + (7x + 2) = 90 \\
11x + 2 = 90 \\
11x + 2 - 2 = 90 - 2 \\
11x = 88 \\
x = 8
\]

Solution: \( x = 8 \)

M 6 Use the diagram and the value of \( x \) to find the measures of \( \angle DOT \) and \( \angle TOU \).

The measure of \( \angle DOT \) = 58°

The measure of \( \angle TOU \) = 32°

M 7 What is the measure of \( \angle SOC \)? Justify your answer.

The measure of \( \angle SOC \) = 58°; \( \angle SOC \) and \( \angle DOT \) are vertical angles, so their measures are the same.

C 8 Tell whether two angles can be as described. Justify your answers.

a. vertical and complementary

Yes; if each of two vertical angles has measure 45°, then they are also complementary.

b. vertical and supplementary

Yes; if each of two vertical angles has measure 90°, then they are also supplementary.

c. complementary and supplementary

No; the sum of the measures of two angles cannot be both 90° and 180°.
Lesson 18

Problem Solving with Angles

Solve the problems.

1. Two angles are vertical angles. One angle is labeled $2x^\circ$. The other angle is labeled $(x + 30)^\circ$. Find the value of $x$.
   - A) 30
   - B) 90
   - C) 120
   - D) 180

2. In the diagram, $\overline{CA}$ and $\overline{BA}$ are extended as shown. Tell whether each statement is True or False.
   - a. $\angle EAF$ and $\angle BAC$ are complementary. $\boxed{\text{False}}$
   - b. $x = 14$. $\boxed{\text{True}}$
   - c. $\angle COB$ and $\angle DOA$ are vertical angles. $\boxed{\text{True}}$
   - d. $\angle BAC$ and $\angle FAE$ are supplementary. $\boxed{\text{False}}$
   - e. $m \angle CAD = 62^\circ$. $\boxed{\text{True}}$
   - f. $\angle EAF$ and $\angle BAC$ are equal in measure. $\boxed{\text{False}}$

3. From the following angle measures, choose a pair that is complementary and a pair that is supplementary.
   - Complementary: 78° and 12°
   - Supplementary: 102° and 78°

4. Solve.
   - a. The hour and minute hands on the clock shown form a straight line. The location of the second hand on the clock is also shown on the diagram. Find the measures, in degrees, of angle $HOS$ and angle $MOS$.
     - Show your work.
     - Solution: The measure of $\angle HOS = (3 \cdot 21 + 5)^\circ = (63 + 5)^\circ = 68^\circ$.
     - The measure of $\angle MOS = (5 \cdot 21 + 7)^\circ = (105 + 7)^\circ = 112^\circ$.

   - b. What does the given information tell you about the angles in the diagram?
     - $x = 15$

   - c. Find $x$:
     - $5x = 21$

   - d. What type of angle forms when the minute and hour hands are in a straight line?
     - $120^\circ$

   - e. In the diagram, $\overline{PM} \parallel \overline{AB}$.
     - a. Explain how $\angle MPD$ and $\angle APC$ are each related to $\angle DPB$.
       - $\angle MPD$ is complementary to $\angle DPB$. $\angle APC$ is vertical and therefore congruent to $\angle DPB$.
     - b. Write an expression that relates the measures of $\angle MPD$ and $\angle APC$ in terms of $x$.
       - Possible answer: $90^\circ - \frac{1}{2}x + 15^\circ = (2x - 50)^\circ$.
     - c. What is the value of $x$?
       - 50

   - f. What does the given information tell you about the angle in the diagram?
     - $2x = 15$