### Lesson 13

#### Proportional Relationships

**Solving Problems with Percents**

Study the example problem showing how to solve a problem with percents. Then solve problems 1–7.

**Example**

Chumani has a coupon for 15% off the total bill at a new restaurant. Her original bill is $32.00. After the discount, what amount is her bill?

1. First, find the amount of the discount.
   \[ 15\% = 0.15 \text{ and } 0.15 \times 32 = 4.80 \]
2. Subtract the discount from the original amount.
   \[ $32.00 - 4.80 = 27.20 \]
3. So Chumani's bill is $27.20.

**B 1**

A bar model can also be used to represent the problem. Complete the bar model.

<table>
<thead>
<tr>
<th>Original Bill</th>
<th>$32.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounted Bill</td>
<td>b</td>
</tr>
<tr>
<td>Amount of Discount</td>
<td>$32</td>
</tr>
</tbody>
</table>

**B 2**

Write and solve an equation to represent the relationship shown in the bar model.

Possible answer: \[ b = 32 - (0.15 \times 32) = 32 - 4.80 = 27.20 \]

**M 3**

The percent of the discount in the problem above is 15%. What percent of the original bill is Chumani’s bill?

- **How could you solve the problem using this percent?**
  - **85%:** The amount Chumani paid is 100% - 15%, or 85%, of the original price.
  - **85% of 32:** \[ 0.85 \times 32 = 27.20 \]

**C 7**

A jacket that originally sold for $60.00 was on sale for 10% off. When it didn’t sell after several weeks, the sale price was discounted another 40% off the discounted price. What was the final price of the jacket? Is the total discount equal to 50%? Explain.

1st discount: \[ 0.10 \times 60 = 6 \]
1st sale price: \[ 60 - 6 = 54 \]
2nd discount: \[ 0.40 \times 54 = 21.60 \]
Final price: \[ 54 - 21.60 = 32.40 \]

No; 50% of $60.00 = $30, but the actual total discount is $27.60.

### Practice

| M 4 | Chander earns a base pay of $2,200 per month. He also earns a commission of 4% of his total sales. One month Chander earned $2,400. Explain how to find Chander’s total sales for that month. How much were his total sales? |
| --- | $5,000; Possible answer: Subtract the base pay from the total pay to find the amount of the commission. Then divide the commission amount by 0.04, which is 4% written as a decimal. $2000 ÷ 0.04 = 20,000 ÷ 4 = 5,000. |
| M 5 | Marian borrowed money to buy a sound system that costs $450. She is charged 5% simple interest for one year. What is the total amount that she pays for the sound system if she pays the full amount in one year? |
| --- | Marian pays a total of $472.50 for the sound system. |
| M 6 | Dan paid $34.30 for a sweater. The price included a 40% markup. Find the cost of the sweater before the markup was added. |
| --- | The cost of the sweater before the markup was $24.50. |
| M 7 | A jacket that originally sold for $60.00 was on sale for 10% off. When it didn’t sell after several weeks, the sale price was discounted another 40% off the discounted price. What was the final price of the jacket? Is the total discount equal to 50%? Explain. |
| --- | No; 50% of $60.00 = $30, but the actual total discount is $27.60. |
Lesson 13

Finding Percent Change

Study the example problem showing how to find percent change. Then solve problems 1–7.

Example

The plants that Loma grew for her science project averaged 6 inches in height. Two weeks later, the plants averaged 9 inches in height. What was the percent increase in the average height of the plants?

You can use a bar model to compare the original height to the change in height.

You can also use the proportion below to compare the change to the original amount.

\[
\frac{\text{amount of change}}{\text{original amount}} = \text{percent change}
\]

\[
\frac{9 - 6}{6} = \frac{3}{6} = \frac{1}{2} = 50\% \\
\frac{1}{2} = 50\% 
\]

1. Use either the bar model or the proportion to solve for \(x\).
   What was the percent increase in the average height of the plants?
   **The percent increase is 50%**. Possible work:

   \[
   3 \times \frac{1}{2} = 50\% \\
   \frac{1}{2} = 50\% 
   \]

2. Would the percent change be greater than or less than 50% if the plants had grown to 8 inches instead of 9?
   **Less than 50%**

3. After 4 weeks, the height of the plants had grown from 6 inches to 12 inches. Write and solve a proportion to find the percent increase in the height of the plants.
   \[
   \frac{12 - 6}{6} = \frac{x}{6} = \frac{6}{6} = 100\% \\
   \text{There was a 100% increase in the height of the plants.}
   \]

4. Students donated 2,500 cans of food to the local food pantry last year. They donated 4,000 cans this year. What is the percent increase in the number of cans donated?
   **Solution:**

   \[
   \frac{4,000 - 2,500}{2,500} = \frac{x}{100} \Rightarrow x = 60
   \]
   **The percent increase is 50%**.

5. Mike plays basketball. He attempted 32 free throws in January and 28 free throws in February.
   a. Is the percent change a percent increase or a percent decrease?
   b. Write and solve a proportion to find the percent change in the number of free throws.
   **Solution:**

   \[
   \frac{32 - 28}{32} = \frac{x}{100} \Rightarrow x = 12.5
   \]
   **The percent decrease is 12.5%**.

6. Find the percent of increase or decrease.
   a. \(x\) to \(5x\) **400% increase**
   b. 2.5\(x\) to 1.5\(x\) **40% decrease**
   c. \(n\) to \(\frac{n}{5}\) **20% decrease**
   d. 3.2\(t\) to 5.2\(t\) **62.5% increase**

7. A store manager pays $40 for a shirt and adds a markup of 20%. During a store sale, the manager marks the cost of the shirt down by 20%. What is the percent of change from the original cost, $40, to the sale price?
   **The marked-up price is 40 \times 1.2 = 48. The sale price is 0.8 \times 48 = 38.40.**
   \[
   \frac{48 - 38.40}{48} = \frac{x}{100} \Rightarrow x = 20\% \\
   \text{the percent decrease is 4%}
   \]

Solve.
Lesson 13

Problem Solving

Percent Error

Study the example problem showing how to find percent error. Then solve problems 1–8.

Example

A thermometer manufacturer compares the reading on one of its thermometers to a thermometer that they know is accurate. The accurate thermometer reads 25°C. Their thermometer reads 30°C. What is the percent error in their thermometer’s reading?

You can use a bar model to help you understand the problem.

You can also use a proportion.

\[
\frac{\text{amount of error}}{\text{actual amount}} = \frac{x}{100}
\]

\[
30 - 25 = \frac{x}{100}
\]

\[
x = 5 \times 100 = 50
\]

\[20\% \text{ possible solution: } \frac{5}{25} = \frac{x}{100} \Rightarrow \frac{1}{5} = \frac{x}{100} \Rightarrow x = 20
\]

Use either the bar model or the proportion to solve for x.

What is the percent error in the thermometer’s reading?

20%; Possible solution: \( \frac{5}{25} = \frac{x}{100} \Rightarrow \frac{1}{5} = \frac{x}{100} \Rightarrow x = 20 \)

Explain the relationship between the \( x \) in the bar model and the \( x \) in the proportion.

They are the same. Both represent writing the ratio 5 to 25 as a percent.

The reading on a different thermometer is 23°C. Do you think the percent error of this thermometer is more or less than for the first thermometer? Explain your answer.

Less; The difference between 23 and 25 is less than the difference between 30 and 25.

Find the percent error of a thermometer reading of 23°C.

8%
Lesson 13

Proportional Relationships

Solve the problems.

1. For each of the following situations, is the percent of change between 20% and 30%? Select Yes or No for each situation.
   a. A $12 cost increases to $15. Yes No
   b. A boy’s height increases from 52 inches to 61 inches. Yes No
   c. The temperature falls from 3°F to 2°F. Yes No

2. A customer pays $18 for a DVD that originally cost $20. What is the percent decrease in the cost of the DVD?
   a. 2%
   b. 10%
   c. 11%
   d. 18%
   Barbara chose c as the correct answer. How did she get that answer?
   Possible answer: She divided the change in the cost, $2, by the new cost, $18. She should have divided by the original cost, $20.

3. Brady needs to cut a piece of scrapbook paper 12 centimeters long. He cuts pieces of the following lengths. Which cut results in a percent error of 15%? Select all that apply.
   a. 10.2 centimeters
   b. 11.2 centimeters
   c. 13.8 centimeters
   d. 18.0 centimeters

4. Jasmine sells beaded jewelry. She calculates the price at which she sells the jewelry by adding a percent markup to the amount it costs her to make the jewelry. Complete the following table. Record money amounts to the nearest cent and markups to the nearest whole percent.

<table>
<thead>
<tr>
<th>Type of Jewelry</th>
<th>Cost to Make ($)</th>
<th>Percent Markup (%)</th>
<th>Selling Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bracelet</td>
<td>7.69</td>
<td>40</td>
<td>10.77</td>
</tr>
<tr>
<td>Necklace</td>
<td>8.66</td>
<td>12</td>
<td>10.77</td>
</tr>
</tbody>
</table>

   Possible work:
   7.69 + 0.4 = 8.09
   8.09 + 0.45 = 8.54
   8.09 + 0.45 = 8.54
   10.77

5. In the late 1980s, there were only 22 California condors, which are large predatory birds, living in the wild. In 2012, the population had increased to 405 condors. To the nearest whole percent, what is the percent increase in the California condor population?
   Solution: To the nearest percent, the percent increase is 1,741%.

6. The manufacturer of an oven states that the temperatures displayed while cooking are within a 5% error. Rachel is using the oven to cook a chocolate cake that must be cooked at a temperature below 325°C. The oven display shows a temperature of 310°C. Can Rachel be certain that the temperature is suitable for cooking her cake? Explain your answer.
   No: The possible error of a 310°C reading is 310 - 30 = 0.6. Because the error may be more than this, the temperature may be too hot for Rachel’s cake.