Lesson 6
Multiply and Divide Rational Numbers

Prerequisite: Multiply Positive and Negative Integers

Study the example problem showing how to use repeated addition to multiply a positive and a negative integer. Then solve problems 1–6.

Example
Dues for the art club are $2 per week. Noriko pays $2 each week from her bank account for her art club dues. At the end of 5 weeks, what is the change in the amount of money in Noriko’s account?

You can think of this as 5 groups of $-2$.
5 groups of $(-2) = (-2) + (-2) + (-2) + (-2) + (-2)$.
You can start at 0 and make 5 jumps of $-2$.

\[ (-2) + (-2) + (-2) + (-2) + (-2) = -10 \]

So, Noriko’s account changes by $-10$.

B 1 Complete the table and describe the pattern in the products.

The products increase by 2.

B 2 Rewrite the repeated addition expression as a multiplication expression and find the product.

a. $(-2) + (-2) + (-2) + (-2) + (-2) = \frac{5}{(-2)} = -10$

b. $(-3) + (-3) + (-3) + (-3) + (-3) = \frac{6}{(-3)} = -18$

c. $(-5) + (-5) + (-5) + (-5) = \frac{4}{(-5)} = -15$

Solve.

M 1 Write and simplify a multiplication expression for each problem. Explain what the product means.

a. The temperature dropped 8 degrees every hour. What was the change in temperature after 4 hours?

\[ 4 \times (-8) = -32; \text{ the temperature is 32 degrees colder.} \]

b. Mae loses 3 points for each wrong answer on a test. How does her score change if she has 7 wrong answers?

\[ 7 \times (-3) = -21; \text{ her test score drops by } 21 \text{ points.} \]

M 2 Carlo is playing a game with the cards shown. He draws cards at random and multiplies the numbers.

a. Give an example of two cards Carlo could draw that have a positive product. Find the product.

Possible answer: $-2 \times (-3) = 6$

b. Give an example of two cards Carlo could draw that have a negative product. Find the product.

Possible answer: $-2 \times 8 = -16$

M 3 Each month Marla pays a $30 cell phone bill from her bank account. At the end of one year, what is the change in the amount of money in Marla’s account because of her cell phone bill? Write a multiplication equation to represent the problem. Explain what the product represents.

\[ -30 \times 12 = -360; \text{ her account balance decreased } \$360. \]

C 4 How many integers must be negative for the product of three integers to be negative? Explain. Find two different groups of three integers whose product is $-12$.

Show your work.

Possible work:

\[ -2 \times (-2) \times (-3) = -12 \]

\[ 6 \times (-2) \times 1 = -12 \]

Solution: Possible answer: $-2$, $-2$, and $-3$; $6$, $-2$, and $1$; Either 1 or 3 of the integers must be negative for the product to be negative. If all three integers were positive or only two of the integers were negative, the product would be positive.
Multiply Rational Numbers

Study the example problem showing how to multiply rational numbers. Then solve problems 1–6.

Example

The temperature of a liquid in an experiment is 0°C at 9 AM. The experiment calls for the temperature to change 4°C every hour. What will the temperature be at noon? The model shows that the temperature will be –2°C, or –3 1/4°C, at noon.

You can also use multiplication to solve:

\[
3 \cdot \left(-\frac{1}{4}\right) = -\frac{15}{4} = -3\frac{3}{4}
\]

**B** 1

If the liquid continues to cool at the same rate, what will the temperature be at 3 PM? Explain. –7 1/2°C; Possible explanation: After 6 hours of cooling, the temperature will be 6 \cdot \left(-\frac{1}{4}\right) = -\frac{36}{4}, or –7 1/2°C.

**B** 2

If the experiment called for the temperature of the liquid to change by –3°C each hour, what would the temperature be at noon? Explain. –2 3/4°C; Possible explanation: 3 \cdot \left(-\frac{3}{4}\right) = -\frac{9}{4}, or –2 3/4°C.

**M** 3

In a different experiment, a liquid must be cooled 6 times as fast as the liquid in the example, but it must still start at 0°C.

a. What will the change in temperature be each hour? Write and evaluate an expression to explain.

\[-\frac{15}{2} °C, or –7 1/2°C; Possible explanation: 6 \cdot \left(-\frac{5}{4}\right) = -\frac{30}{4} = -\frac{15}{2}\]

b. What will the temperature be after 4 hours? Write and evaluate an expression to explain.

\[-30°C; 4 \cdot \left(-\frac{15}{2}\right) = -\frac{60}{2} = -30\]

Solve.

**M** 4

Evaluate the products.

a. \(4 \times \left(-\frac{3}{8}\right) = -\frac{3}{2} \text{ or } -1\frac{1}{2}\)

b. \((-4) \times \left(-\frac{1}{2}\right) = \frac{2}{2} \text{ or } 1\)

**B** 5

How can you use the rules for multiplying positive and negative numbers to check that your answers to problem 4 have the correct sign?

The product of a positive number and a negative number is negative. The product of two negative numbers is positive.

**M** 6

A deep sea diver changes his position relative to the surface of the water by –7/2 meters per minute. Write and evaluate an expression to show his position after 3 minutes if he starts at the surface of the water.

\[3 \times \left(-\frac{7}{2}\right) = -\frac{21}{2}, \text{ or } –10\frac{1}{2} \text{ meters}\]

**M** 7

Evaluate the products. Write your answers in simplest form.

a. \(-\frac{4}{3} \times \left(-\frac{9}{8}\right) = \frac{1}{2}\)

b. \(-\frac{3}{10} \times \left(-\frac{2}{3}\right) = \frac{3}{5}\)

**C** 8

The height of a 9-inch candle changes at a rate of –2 1/2 inch per hour when it is burning. How long will it take the candle to burn completely?

Show your work.

Possible work:

I have to answer the problem \(? \times \left(-\frac{2}{3}\right) = -9\). To do this, I can guess and check.

\[13 \times \left(-\frac{2}{3}\right) = -\frac{26}{3} = -8\frac{2}{3}\]

\[14 \times \left(-\frac{2}{3}\right) = -\frac{28}{3} = -9\frac{1}{3}\]

Because \(-9 = -\frac{27}{3}\) and \(-27\) is halfway between \(-26\) and \(-28\), I know that it will take halfway between 13 and 14 hours to burn completely.

Solution: It will take 13 1/2 hours for the candle to burn completely.
Divide Rational Numbers

Study the example problem showing how to divide rational numbers. Then solve problems 1–6.

Example

The position of a submersible water vehicle, relative to sea level, changes by −0.5 mile each hour. After how many hours will the vehicle’s position have changed by −2.5 miles?

You can use a table to understand the problem.

You can also use division to understand the problem.

\[(−2.5) ÷ (−0.5) = 5\]

You can also use repeated addition to evaluate the quotient (−2.5) ÷ (−0.5). Complete.

\[−2.5 = (−0.5) + (−0.5) + (−0.5) + (−0.5) + (−0.5)\]

There are 5 groups of −0.5 in −2.5.

It will take 5 hours for the submersible’s position to change by −2.5 miles.

Mariella’s bank statement shows a change of −$2.50 in her account each week.

a. Complete the table to see how many weeks it will be before the change in Mariella’s account is −$20.

<table>
<thead>
<tr>
<th>Number of Weeks</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in Account ($)</td>
<td>−2.5</td>
<td>−5.0</td>
<td>−7.5</td>
<td>−10.0</td>
<td>−12.5</td>
<td>−15.0</td>
<td>−17.5</td>
<td>−20.0</td>
</tr>
</tbody>
</table>

b. Why are the amounts in the table negative?

Money is being removed from Mariella’s account, not added to it.

c. How many weeks will it take for the amount deducted to total −$20? 8 weeks

B 1

Solve.

B 2

A pitcher contains 28 fluid ounces of juice. How many 4-ounce servings can you pour before the pitcher is empty? Complete the repeated subtraction to solve.

\[−28 = (−4) + (−4) + (−4) + (−4) + (−4) + (−4)\]

I can pour 7 four-ounce glasses of juice.

B 3

A small inflatable pool holds 10 gallons of water. A tiny leak causes the amount of water in the pool to change by −0.05 gallon each hour until the pool is empty.

a. Write the numbers that represent each quantity.

change in amount of water per hour = −0.05 gallon

total change in amount of water = −10 gallons

b. Will you multiply or divide to find how many hours it will take for the pool to be empty? Will the answer be positive or negative?

\[
\text{divide; positive; The quotient of two negative numbers is positive; also, the number of hours the pool is leaking is a positive number.}
\]

c. How many hours will it take for the pool to be empty? 200 hours

M 4

The temperature at 6 PM was 0°F. At 10 PM the temperature was −11.2°F. Write an expression that you can use to find the average change in temperature per hour during that time. Then evaluate the expression.

\[−11.2 − 4 = −2.8\]

C 5

Marielle is solving a number puzzle involving three rational numbers a, b, and c. Here are her clues:

• The quotient of a and c is 8.
• The product of a and c is 2.
• The quotient of a and b is −16.
• The product of a and b is −1.
• The quotient of b and c is −0.5.

What are the three numbers in Marielle’s puzzle?

\[a = −4, b = 0.25, c = −0.5\text{ or }a = 4, b = −0.25, c = 0.5\]
Lesson 6

Multiply and Divide Rational Numbers

Solve the problems.

1. Imamu pays $30 each month for his gym membership. What is the change in the amount of money in his account after \( \frac{1}{2} \) of a year?
   - A $120
   - B $240
   - C $270
   - D $360
   Bob chose A as the correct answer. How did he get that answer?
   Bob found the change in the amount of money after \( \frac{1}{2} \) of a year instead of \( \frac{2}{3} \) of a year.

2. Trevon, Leah, and Beth are playing a computer game. Trevon scored \(-8\). Beth’s score was \( \frac{3}{4} \) of Trevon’s score, and Leah’s score was \( \frac{1}{3} \) of Beth’s score. What was Leah’s score?
   - A \( -\frac{2}{3} \)
   - B \( -\frac{3}{2} \)
   - C \( \frac{2}{3} \)
   - D \( \frac{3}{2} \)

3. Paula cut ribbon from a spool for a craft project. Each piece of ribbon changed the length of the ribbon remaining on the spool by \(-1.2\) feet. When Paula finished, the length of ribbon on the spool had changed by \(-9.6\) feet. How many pieces of ribbon did Paula cut?
   Show your work.
   Possible work: \( \frac{3}{5} = 0.6 \), so \( -9.6 \times 5 = -9.6 \)
   \(-9.6 = (-1.2) + (-1.2) + (-1.2) + (-1.2) + (-1.2) + (-1.2) + (-1.2) + (-1.2) \)
   \(-9.6 + (-1.2) = 8 \)
   Solution: Paula cut 8 pieces of ribbon.

4. Craig is making 3 recipes for his party. He has a container of flour on his kitchen scale. The table shows the total change in weight of the flour on the scale after each of the 3 recipes. What is the average change in the amount of flour, in ounces, used in the 3 recipes?
   
<table>
<thead>
<tr>
<th>Change in Weight (oz)</th>
<th>After Recipe 1</th>
<th>After Recipe 2</th>
<th>After Recipe 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-3)</td>
<td>(-2)</td>
<td>(-1)</td>
</tr>
<tr>
<td>B</td>
<td>(-\frac{3}{2})</td>
<td>(-\frac{5}{2})</td>
<td>(-\frac{7}{2})</td>
</tr>
<tr>
<td>C</td>
<td>(-\frac{1}{2})</td>
<td>(-\frac{3}{2})</td>
<td>(-\frac{5}{2})</td>
</tr>
<tr>
<td>D</td>
<td>(-\frac{3}{2})</td>
<td>(-\frac{5}{2})</td>
<td>(-\frac{7}{2})</td>
</tr>
</tbody>
</table>

5. On the number line, \( a \) and \( b \) are rational numbers. Tell whether each statement is True or False.
   - a. \( a \cdot b \) is negative.
   - b. \( a \cdot b \) is greater than 0.
   - c. \( b - a \) is greater than \( a - b \).
   - d. \( a \cdot b = b \cdot a \)
   - e. \( a \cdot b = b \cdot a \)

6. Martin chose two of the cards below. When he found the quotient of the numbers, his answer was \(-\frac{16}{9}\). Write the division problem that Martin solved.
   Show your work.
   Possible work: \( \frac{3}{2} \div \frac{-3}{8} = \frac{2}{3} \times \frac{8}{3} = -\frac{16}{9} \)
   Solution: \( \frac{3}{2} \div \frac{-3}{8} \)